DP Analysis & Approaches SL / HL

Maths with Lemon

Financial Applications: Guided Notes and Worksheet

Mathematician: _____

Learning objectives

Model percentage increase and decrease using geometric sequences; calculate future and past values; solve for time; use compound interest with different compounding periods; model regular deposits and withdrawals; distinguish nominal and real values.

1. Percentage change and geometric growth

For an initial value PV, rate $r\%$ per period, and n complete periods:

$$\boxed{FV = PV\left(1+\frac{r}{100}\right)^n} \quad \text{(growth)}, \qquad \boxed{FV = PV\left(1-\frac{r}{100}\right)^n} \quad \text{(decrease)}.$$

The multiplier R is $1 + r/100$ for growth and $1 - r/100$ for decrease. Money is normally rounded to two decimal places.

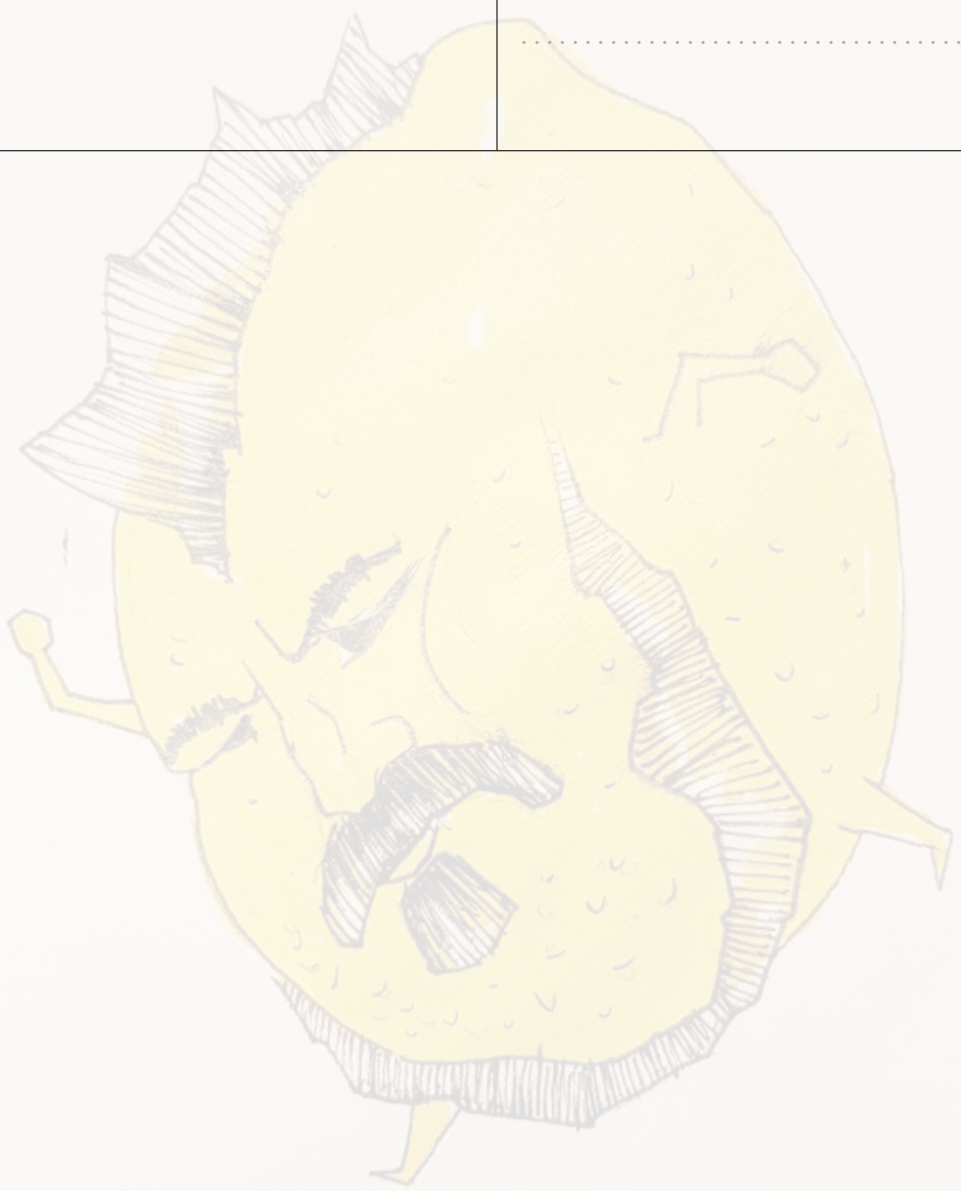
Rate	Increase multiplier	Decrease multiplier
12%	1.12	0.88
20%	1.20	0.80
5%	1.05	0.95

Worked Example	Your Turn
Example 1. Invest €2000 at 8% annually for 10 years. Compare with depreciation at 8%. $FV_{\text{growth}} = 2000(1.08)^{10} \approx \boxed{4317.85}$. $FV_{\text{decrease}} = 2000(0.92)^{10} \approx \boxed{868.78}$.	**Practice 1.** A laptop costs €1500 and loses 12% of its value each year. Find its value after 5 years. . . . .

2. Geometric sequences and the starting date

If the *first term* is the amount in year 1, then $u_n = u_1 R^{n-1}$. If the initial amount is given *today*, after n complete years $FV = PVR^n$. Count elapsed periods carefully.

Worked Example	Your Turn
Example 2. €5000 is invested at the beginning of 2024 at 8% per year. Find the value at the beginning of 2031. There are $2031 - 2024 = 7$ periods. $FV = 5000(1.08)^7 \approx \boxed{8569.12}$.	**Practice 2.** A fund contains €3200 at the beginning of 2025. It grows by 6% per year. Find its value at the beginning of 2032. ....



3. Finding the number of complete years

To find the first complete year when a threshold is exceeded, solve $PVR^n > T$ using logarithms or a GDC, then round *up* to the next integer. Check the two adjacent integer years.

Worked Example	Your Turn
Example 3. €2000 grows by 8% per year. When will it first exceed €5000? $2000(1.08)^n > 5000 \Rightarrow n > \dfrac{\ln(2.5)}{\ln(1.08)} \approx 11.91.$ So it first exceeds the target after $\boxed{12 \text{ complete years}}$.	**Practice 3.** An investment of €3500 grows by 5% annually. Find the first complete year when its value exceeds €6000. . . . . .

4. Population growth: future, past, and doubling

For n years in the past, use $PV\,R^{-n} = PV/R^n$. For doubling, solve $R^n = 2$.

Worked Example	Your Turn
Example 4. A city has 800,000 people and grows by 5.2% annually. After 7 years: $800000(1.052)^7 \approx \boxed{1\,140\,775}$. Seven years ago: $800000(1.052)^{-7} \approx \boxed{561\,022}$. Doubling time: $n = \dfrac{\ln 2}{\ln 1.052} \approx 13.68,$ so $\boxed{14 \text{ complete years}}$.	**Practice 4.** A town has 120,000 people and grows by 3% annually. Find (a) its population after 8 years, (b) its population 8 years ago, and (c) the first complete year in which it doubles. . . . . .

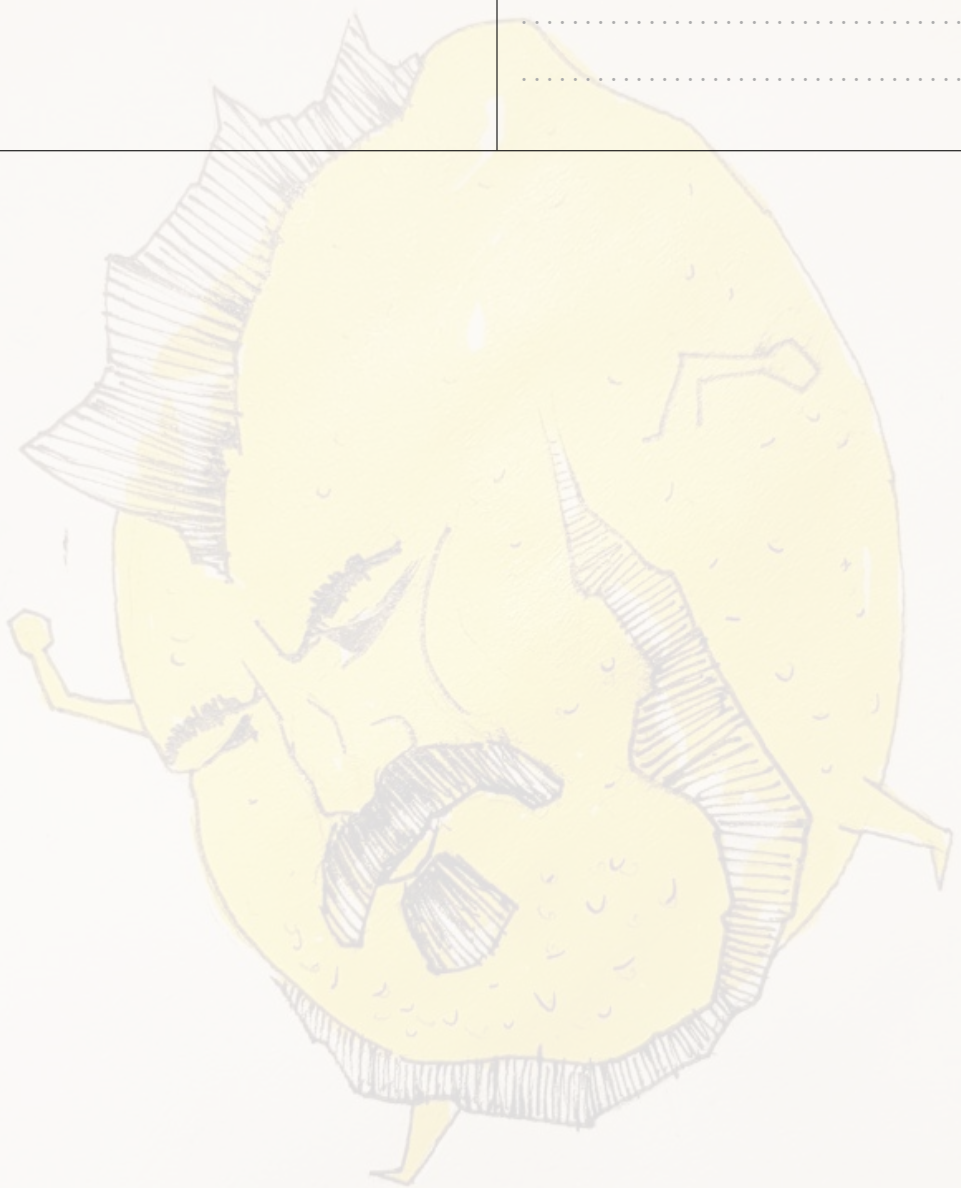
5. ~~Compound interest~~ more than once per year

For nominal annual interest rate $r\%$, compounded k times each year, over t years:

$$\boxed{FV = PV\left(1 + \frac{r}{100k}\right)^{kt}}.$$

$k = 1$ yearly, 2 semiannually, 4 quarterly, 12 monthly.

Worked Example	Your Turn
Example 5. Invest €1000 at 12% nominal interest for 5 years. Yearly: $1000(1.12)^5 \approx \boxed{1762.34}$. Monthly: $1000(1.01)^{60} \approx \boxed{1816.70}$. More frequent compounding gives a larger value for a positive nominal rate.	**Practice 5.** Invest €2500 at 6% nominal annual interest for 4 years. Calculate the final amount if interest is compounded (a) annually, (b) quarterly, and (c) monthly. ....



6. Regular end-of-period deposits

For annual compounding and equal deposits PMT at the *end* of each year, $R = 1 + r/100$ and

$$\boxed{FV = PV\,R^n + PMT\left(\frac{R^n - 1}{R - 1}\right)}.$$

The final deposit is included. In a financial solver, deposits are usually entered as negative cash flows, while the final balance is positive. For $R = 1$, the deposit term becomes $n\,PMT$.

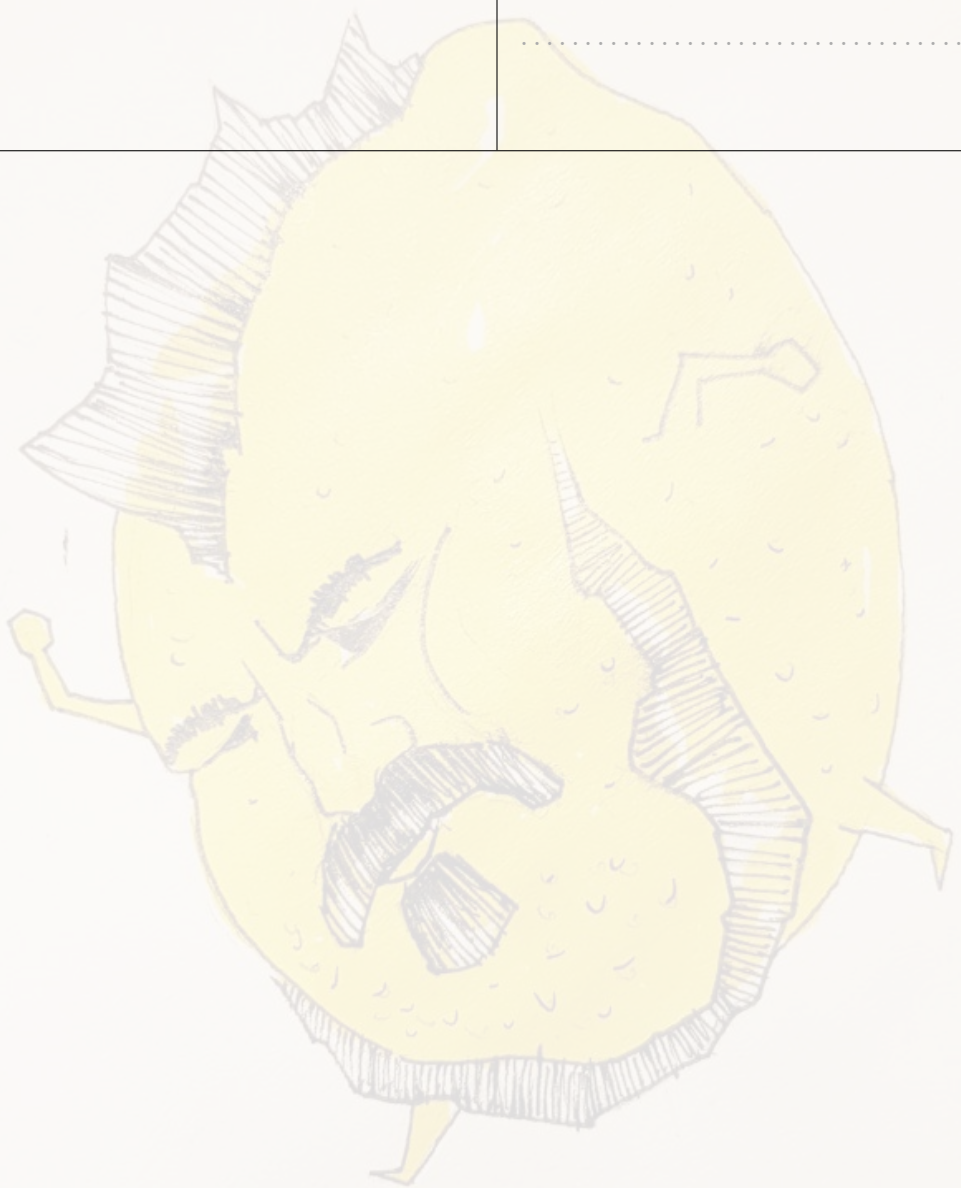
Worked Example	Your Turn
Example 6. Start with €1000 at 12% annual interest. Add €300 at the end of each year for 7 years. $$FV = 1000(1.12)^7 + 300\frac{(1.12)^7 - 1}{0.12}$$ $\boxed{FV \approx €5237.38}$. The last €300 is included.	**Practice 6.** Invest €2000 at 5% annual interest and add €400 at the end of each year for 6 years. Find the final balance. . . . .

7. Monthly compounding with annual or monthly deposits

If interest compounds monthly but deposits occur annually, use the *effective annual multiplier* $R = (1 + r/1200)^{12}$ in the annual-deposit formula. If deposits are monthly, use the monthly multiplier $R = 1 + r/1200$ and $n = 12t$.

Worked Example	Your Turn
Example 7. Start with €1000 at 12% nominal interest compounded monthly. Deposit €300 at the end of each *year* for 7 years. Let $R = (1.01)^{12}$. Then $$FV = 1000R^7 + 300\frac{R^7 - 1}{R - 1}$$ $\boxed{FV \approx €5397.73}$.	**Practice 7.** Start with €1500 at 6% nominal interest compounded monthly. Deposit €250 at the end of each year for 5 years. Find FV. . . . .

Worked Example	Your Turn
Example 8. Keep the same initial €1000 and 12% nominal monthly-compounded interest, but deposit €300 at the end of *each month* for 7 years. $R = 1.01$ and $n = 84$. $$FV = 1000(1.01)^{84} + 300\frac{(1.01)^{84} - 1}{0.01}$$ $\boxed{FV \approx €41508.41}$.	**Practice 8.** Invest €1500 at 6% nominal interest compounded monthly. Deposit €250 at the end of *each month* for 5 years. Find FV. . . . .



8. Regular withdrawals and amortization

For withdrawals W at the end of each period, subtract their accumulated value:

$$\boxed{FV = PV\,R^n - W\left(\frac{R^n - 1}{R - 1}\right)}.$$

To determine when a fund is exhausted, solve $FV = 0$. If the final scheduled withdrawal exceeds the balance, the last actual withdrawal must be smaller.

Worked Example	Your Turn
Example 9. Invest €1000 at 12% nominal interest compounded monthly, and withdraw €150 at the end of each year. Set $R = (1.01)^{12}$. After 7 years: $$FV = 1000R^7 - 150\frac{R^7 - 1}{R - 1}$$ $$\boxed{FV \approx €761.22}.$$ The account is depleted during the 16th annual withdrawal.	**Practice 9.** Invest €3000 at 5% interest compounded annually and withdraw €500 at the end of each year. (a) Find the balance after 5 years. (b) Determine the first year in which a full €500 withdrawal is not possible. . . . . .

9. Inflation and real value

If the investment grows at r% per year and inflation is a% per year, the *exact* purchasing-power value after n years is

$$\boxed{RV = \frac{PV(1 + r/100)^n}{(1 + a/100)^n}}.$$

Subtracting the rates, $r - a$, is only an approximation, not an exact formula.

Worked Example	Your Turn
Example 10. Invest €1000 for 10 years at 5% interest with 2% annual inflation. Nominal future value: $$FV = 1000(1.05)^{10} \approx 1628.89.$$ Real value: $$RV = \frac{1000(1.05)^{10}}{(1.02)^{10}} \approx \boxed{1336.26}.$$ Approximation $1000(1.03)^{10} \approx 1343.92$.	**Practice 10.** Invest €4000 for 8 years at 7% interest. Inflation is 3% per year. Find (a) the nominal future value, (b) the exact real value, and (c) the approximate real value using the difference in rates. . . . . .

Calculator financial mode: essential inputs

Input	Meaning
N	Number of payment periods (calculator conventions may vary)
$I\%$	Nominal annual interest rate
PV	Present value
PMT	Regular payment or withdrawal
FV	Future value
P/Y	Payments per year
C/Y	Compounding periods per year

Sign convention: deposits paid into an account are negative cash flows; withdrawals received are positive. Verify payment timing is *END* mode.

Detailed Analytical Markscheme

Practice 1 – Depreciation

$FV = 1500(0.88)^5 = \boxed{€791.60}$.

Practice 2 – Elapsed time

$2032 - 2025 = 7$ years. $FV = 3200(1.06)^7 = \boxed{€4811.62}$.

Practice 3 – First complete year

$$3500(1.05)^n > 6000 \quad \Rightarrow \quad n > \frac{\ln(6000/3500)}{\ln 1.05} \approx 11.05.$$

Thus $\boxed{12 \text{ complete years}}$. Check $n = 11$ is below the target and $n = 12$ exceeds it.

Practice 4 – Population

(a) $120000(1.03)^8 \approx \boxed{152012}$ people.

(b) $120000(1.03)^{-8} \approx \boxed{94729}$ people.

(c) $n \geq \ln 2 / \ln 1.03 \approx 23.45$, so $\boxed{24 \text{ complete years}}$.

Practice 5 – Compounding frequency

(a) $2500(1.06)^4 \approx \boxed{€3156.19}$.

(b) $2500(1 + 0.06/4)^{16} \approx \boxed{€3172.46}$.

(c) $2500(1 + 0.06/12)^{48} \approx \boxed{€3176.22}$.

Practice 6 – Annual deposits

$$FV = 2000(1.05)^6 + 400\frac{(1.05)^6 - 1}{0.05} \approx \boxed{€5400.96}.$$

Practice 7 – Monthly compounding, annual deposits

Set $R = (1.005)^{12}$. Then

$$FV = 1500R^5 + 250\frac{R^5 - 1}{R - 1} \approx \boxed{€3437.28}.$$

Practice 8 – Monthly deposits

$R = 1.005$ and $n = 60$:

$$FV = 1500(1.005)^{60} + 250\frac{(1.005)^{60} - 1}{0.005} \approx \boxed{€19465.78}.$$

Practice 9 – Withdrawals

(a) $FV = 3000(1.05)^5 - 500\dfrac{(1.05)^5 - 1}{0.05} \approx \boxed{€1066.03}$.

(b) Solve $3000(1.05)^n - 500\dfrac{(1.05)^n - 1}{0.05} = 0$:

$$(1.05)^n = \frac{500}{500 - 3000(0.05)} = \frac{10}{7}, \qquad n = \frac{\ln(10/7)}{\ln 1.05} \approx 7.31.$$

Seven full withdrawals are possible; the eighth would exceed the balance. Hence $\boxed{\text{year } 8}$.

Practice 10 – Inflation

(a) $FV = 4000(1.07)^8 \approx \boxed{€6872.74}$.

(b) $RV = 4000(1.07/1.03)^8 \approx \boxed{€5425.41}$.

(c) $RV_{\text{approx}} = 4000(1.04)^8 \approx \boxed{€5474.28}$.

Marking guidance

For each question, award credit for choosing the correct model, identifying the multiplier and number of periods, correct substitution, and a properly rounded final answer with units. For time-to-threshold questions, credit the inequality and rounding up to the first valid complete period. For regular payments, check whether the payments occur annually or monthly and whether the final payment is included.



Maths with Lemon

Mathematics made clear.